Text statistics

September 12, 2023

(Chapter 5.1 IIR)

Overview

1. Term statistics

2. Zipf's law

Outline

1. Term statistics

2. Zipf's law

The Reuters collection

symbol	statistic	value
N	documents	800,000
L	avg. # word tokens per document	200
M	word types	400,000
	avg. # bytes per word token (incl. spaces/punct.)	6
	avg. # bytes per word token (without spaces/punct.)	4.5
	avg. # bytes per word type	7.5
T	non-positional postings	100,000,000

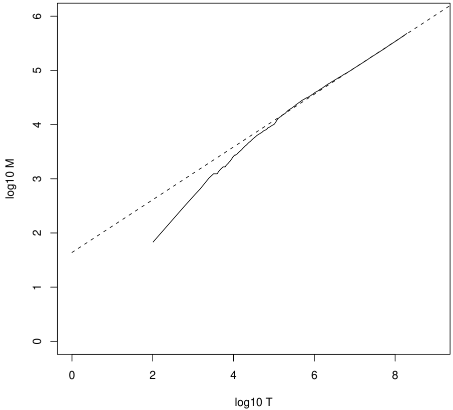
How big is the term vocabulary?

- That is, how many distinct words are there?
- Can we assume there is an upper bound?
- Not really: At least $70^{20} \approx 10^{37}$ different words of length 20.
- The vocabulary will keep growing with collection size.
- Heaps' law:

$$M = kT^b \tag{1}$$

- M is the size of the vocabulary, T is the number of tokens in the collection.
- Typical values for the parameters k and b are: $30 \leq k \leq 100$ and $b \approx 0.5$.
- Heaps' law is linear in log-log space.
  - It is the simplest possible relationship between collection size and vocabulary size in log-log space.
  - Empirical law

Heaps' law for Reuters



Vocabulary size M as a function of collection size T (number of tokens)

$$M = kT^b \tag{2}$$

the best least squares fit for Reuters-RCV1.

$$\log_{10} M = 0.49 * \log_{10} T + 1.64 \tag{3}$$

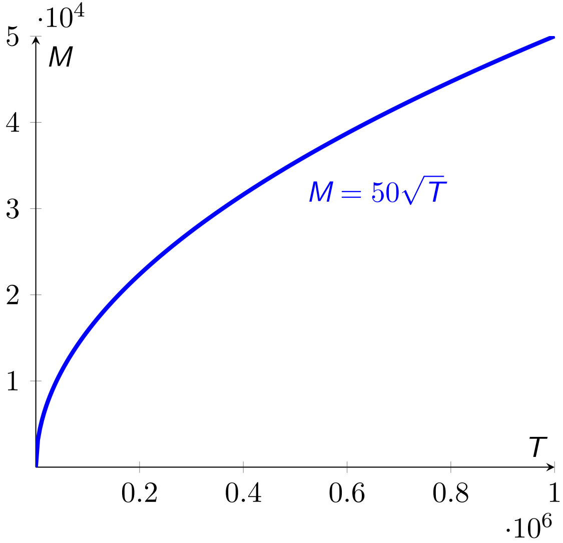
In other words,

$$M = 10^{1.64} T^{0.49} \tag{4}$$

$$k = 10^{1.64} \approx 44 \tag{5}$$

$$b = 0.49 \tag{6}$$

Heaps' law without loglog scale



Empirical fit for Reuters

- Example: for the first 1,000,020 tokens Heaps' law predicts 38,323 terms:

$$44 \times 1{,}000{,}020^{0.49} \approx 38{,}323$$

- The actual number is 38,365 terms, very close to the prediction.
- Empirical observation: fit is good in general.

Exercise

1. What is the effect of including spelling errors vs. automatically correcting spelling errors on Heaps' law?
2. Compute vocabulary size *M*
   - Looking at a collection of web pages, you find that there are 3000 different terms in the first 10,000 tokens and 30,000 different terms in the first 1,000,000 tokens.
   - Assume a search engine indexes a total of 20,000,000,000 (2×10^{10}) pages, containing 200 tokens on average per page
   - What is the size of the vocabulary of the indexed collection as predicted by Heaps' law?

Outline

1. Term statistics

2. Zipf’s law

Zipf's law

- Now we have characterized the growth of the vocabulary in collections.
- We also want to know how many frequent vs. infrequent terms we should expect in a collection.
- In natural language, there are a few very frequent terms and very many very rare terms.
- Zipf's law: The i^{th} most frequent term has frequency cf_i proportional to $1/i$.

$$\mathrm{cf}_i \propto \frac{1}{i} \tag{7}$$

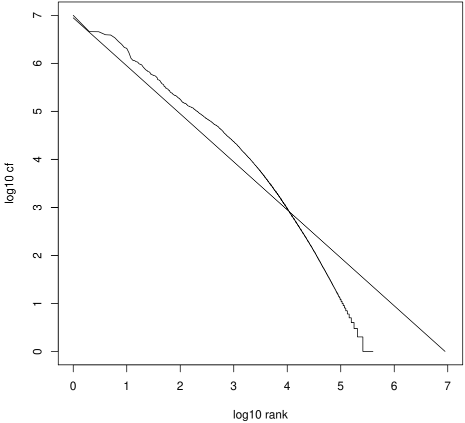
- cf_i is collection frequency: the number of occurrences of the term t_i in the collection.

Zipf’s law

$$\mathrm{cf}_i \propto \frac{1}{i} \tag{8}$$

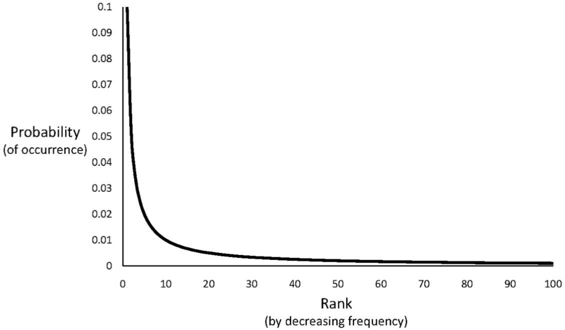
- If the most frequent term (*the*) occurs cf_1 times, then the second most frequent term (*of*) has half as many occurrences $\mathrm{cf}_2 = \frac{1}{2}\mathrm{cf}_1$ ...
- ...and the third most frequent term (*and*) has a third as many occurrences $\mathrm{cf}_3 = \frac{1}{3}\mathrm{cf}_1$ etc.
- Equivalent: $\mathrm{cf}_i = ci^k$ and $\log \mathrm{cf}_i = \log c + k \log i$ (for $k = -1$)
- Example of a power law

Zipf’s law for Reuters

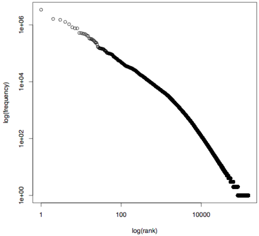


Fit is not great. What is important is the key insight: Few frequent terms, many rare terms.

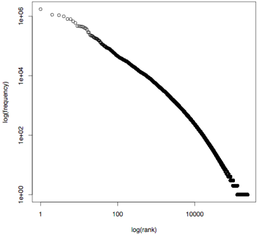
Zipf's law if not log-loged



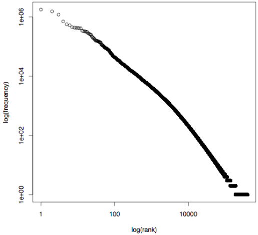
Zipf's law in different languages (parliament documents)



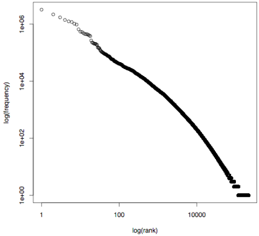
English



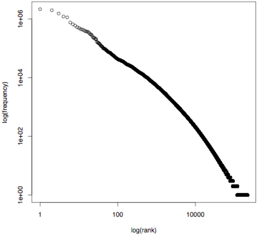
italian



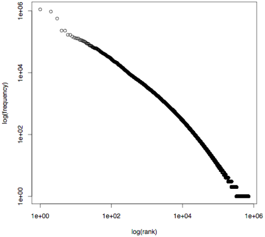
germany



spanish

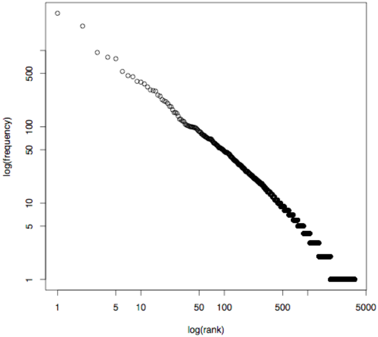


portuguese

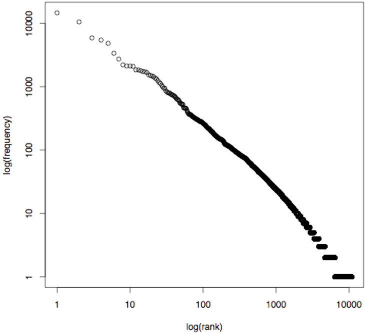


finnish

Zipf's law in scientific writing

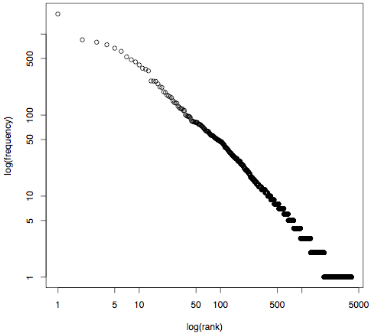


relativity by einstein

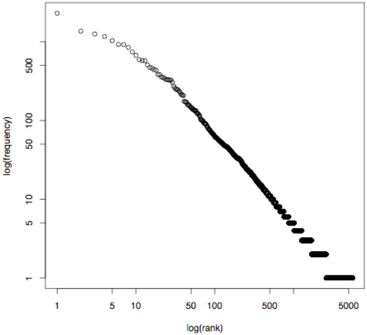


on the origin of species by charles darwin

Zipf's law for kid's books

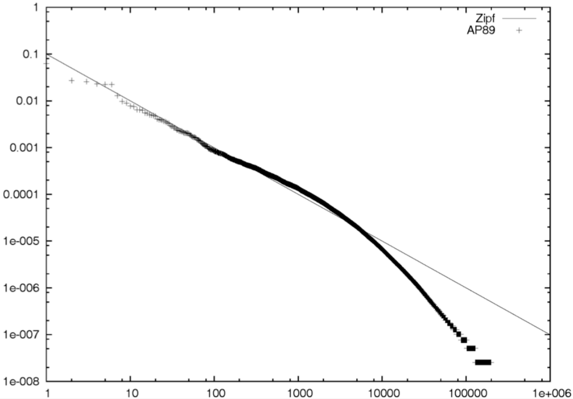


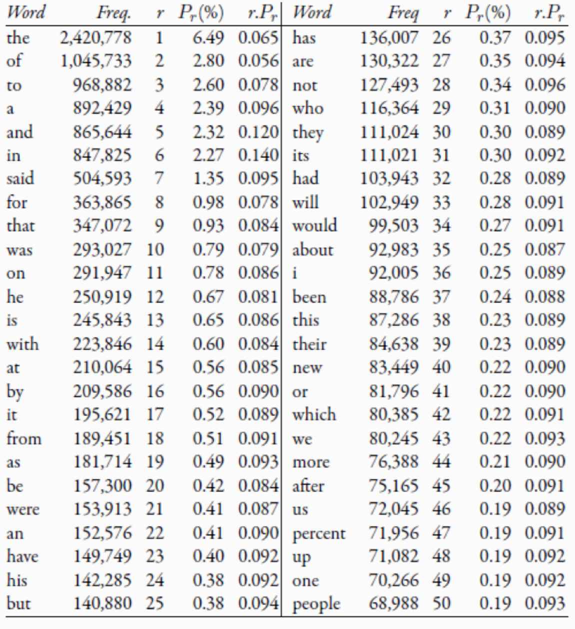
alice



peterpan

Rank	*Word*	*Frequency*
1000	concern	5,100
1001	spoke	5,100
1002	summit	5,100
1003	bring	5,099
1004	star	5,099
1005	immediate	5,099
1006	chemical	5,099
1007	african	5,098





Word	*Freq.*	r	$P_r(\%)$	$r.P_r$	*Word*	*Freq*	r	$P_r(\%)$	$r.P_r$
the	2,420,778	1	6.49	0.065	has	136,007	26	0.37	0.095
of	1,045,733	2	2.80	0.056	are	130,322	27	0.35	0.094
to	968,882	3	2.60	0.078	not	127,493	28	0.34	0.096
a	892,429	4	2.39	0.096	who	116,364	29	0.31	0.090
and	865,644	5	2.32	0.120	they	111,024	30	0.30	0.089
in	847,825	6	2.27	0.140	its	111,021	31	0.30	0.092
said	504,593	7	1.35	0.095	had	103,943	32	0.28	0.089
for	363,865	8	0.98	0.078	will	102,949	33	0.28	0.091
that	347,072	9	0.93	0.084	would	99,503	34	0.27	0.091
was	293,027	10	0.79	0.079	about	92,983	35	0.25	0.087
on	291,947	11	0.78	0.086	i	92,005	36	0.25	0.089
he	250,919	12	0.67	0.081	been	88,786	37	0.24	0.088
is	245,843	13	0.65	0.086	this	87,286	38	0.23	0.089
with	223,846	14	0.60	0.084	their	84,638	39	0.23	0.089
at	210,064	15	0.56	0.085	new	83,449	40	0.22	0.090
by	209,586	16	0.56	0.090	or	81,796	41	0.22	0.090
it	195,621	17	0.52	0.089	which	80,385	42	0.22	0.091
from	189,451	18	0.51	0.091	we	80,245	43	0.22	0.093
as	181,714	19	0.49	0.093	more	76,388	44	0.21	0.090
be	157,300	20	0.42	0.084	after	75,165	45	0.20	0.091
were	153,913	21	0.41	0.087	us	72,045	46	0.19	0.089
an	152,576	22	0.41	0.090	percent	71,956	47	0.19	0.091
have	149,749	23	0.40	0.092	up	71,082	48	0.19	0.092
his	142,285	24	0.38	0.092	one	70,266	49	0.19	0.092
but	140,880	25	0.38	0.094	people	68,988	50	0.19	0.093

Zipf's law is a type of power law

- degree distribution of
  - the web,
  - online social networks,
  - citation networks
  - software networks
- wealth distribution
- there are variants of the power law, such as Mandelbrot law.