naive bayes document classification

October 19, 2017

Overview

Take-away today

- Text classification: definition & relevance to information retrieval
- Naive Bayes: simple baseline text classifier
- Theory: derivation of Naive Bayes classification rule & analysis
- Evaluation of text classification: how do we know it worked / didn't work?

Outline

A text classification task: Email spam filtering

```
From: ``'' <takworlld@hotmail.com>
Subject: real estate is the only way... gem  oalvgkay
Anyone can buy real estate with no money down
Stop paying rent TODAY !
There is no need to spend hundreds or even thousands for similar courses
I am 22 years old and I have already purchased 6 properties using the
methods outlined in this truly INCREDIBLE ebook.
Change your life NOW !
=================================================
Click Below to order:
http://www.wholesaledaily.com/sales/nmd.htm
=================================================
```

How would you write a program that would automatically detect and delete this type of message?

Formal definition of Text Classification: Training

Given:

- A document space $\mathbb{X}$
  - Documents are represented in this space –typically some type of high-dimensional space.
- A fixed set of classes $\mathbb{C} = \{c_1, c_2, \ldots, c_J\}$
  - The classes are human-defined for the needs of an application (e.g., spam vs. nonspam).
- A training set $\mathbb{D}$ of labeled documents. Each labeled document $\langle d, c \rangle \in \mathbb{X} \times \mathbb{C}$

Using a learning method or learning algorithm, we then wish to learn a classifier γ that maps documents to classes:

$$\gamma : \mathbb{X} \to \mathbb{C}$$

Formal definition of Text Classification: Application/Testing

Given: a description $d \in \mathbb{X}$ of a document

Determine: $\gamma(d) \in \mathbb{C}$, i.e., the class that is most appropriate for d

Exercise

- Find examples of uses of text classification in information retrieval

Examples of how search engines use classification

- Language identification (classes: English vs. French etc.)
- The automatic detection of spam pages (spam vs. nonspam)
- Sentiment detection: is a movie or product review positive or negative (positive vs.negative)
- Topic-specific or *vertical* search – restrict search to a "vertical" like "related to health" (relevant to vertical vs. not)
    - Vertical search: focus on a narrow segment of web content

Classification methods: 1. Manual

- Manual classification was used by Yahoo in the beginning of the web. Also: ODP, PubMed
- Very accurate if job is done by experts
- Consistent when the problem size and team is small
- Scaling manual classification is difficult and expensive.
- $\rightarrow$ We need automatic methods for classification.

Classification methods: 2. Rule-based

- E.g., Google Alerts is rule-based classification.
- There are IDE-type development environments for writing very complex rules efficiently. (e.g., Verity)
- Often: Boolean combinations (as in Google Alerts)
- Accuracy is very high if a rule has been carefully refined over time by a subject expert.
- Building and maintaining rule-based classification systems is cumbersome and expensive.

A Verity topic (a complex classification rule)

comment line	`# Beginning of art topic definition`
top-level topic	`art ACCRUE`
topic definition modifiers	`/author = "fsmith"`
	`/date = "30-Dec-01"`
	`/annotation = "Topic created by fsmith"`
subtopictopic	`* 0.70 performing-arts ACCRUE`
evidencetopic	`** 0.50 WORD`
topic definition modifier	`/wordtext = ballet`
evidencetopic	`** 0.50 STEM`
topic definition modifier	`/wordtext = dance`
evidencetopic	`** 0.50 WORD`
topic definition modifier	`/wordtext = opera`
evidencetopic	`** 0.30 WORD`
topic definition modifier	`/wordtext = symphony`
subtopic	`* 0.70 visual-arts ACCRUE`
	`** 0.50 WORD`
	`/wordtext = painting`
	`** 0.50 WORD`
	`/wordtext = sculpture`

```
subtopic    * 0.70 film ACCRUE
            ** 0.50 STEM
                /wordtext = film
subtopic    ** 0.50 motion-picture PHRASE
            *** 1.00 WORD
                /wordtext = motion
            *** 1.00 WORD
                /wordtext = picture
            ** 0.50 STEM
                /wordtext = movie
subtopic    * 0.50 video ACCRUE
            ** 0.50 STEM
                /wordtext = video
            ** 0.50 STEM
                /wordtext = vcr
            # End of art topic
```

Classification methods: 3. Statistical/Probabilistic

- This is our definition of the classification problem – text classification as a learning problem
  - Supervised learning of a the classification function γ
  - application of γ to classifying new documents
- Starting point: Naive Bayes
- Requires training data

Outline

The Naive Bayes text classifier

- The Naive Bayes classifier is a probabilistic classifier.
- Compute the probability of a document d being in a class c:

$$P(c|d) \propto P(c) \prod_{1 \leq k \leq n_d} P(t_k|c)$$

- n_d: the length of the document. (number of tokens).
  - note that it is NB multinomial model. NB Bernoulli differs slightly
- $P(t_k|c)$: conditional probability of term t_k occurring in a document of class c
  - $P(t_k|c)$ as a measure of how much evidence t_k contributes that c is the correct class.
- $P(c)$ is the prior probability of c.
  - If a document's terms do not provide clear evidence for one class vs. another, we choose the c with highest $P(c)$.
  - Naive Bayes for Text Classification with Unbalanced Classes, Eibe Frank, Remco R. Bouckaert, 2006.

Maximum a posteriori class

- Our goal in Naive Bayes classification is to find the "best" class.
- The best class is the most likely or maximum a posteriori (MAP) class c_{map}:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} \hat{P}(c|d) = \arg\max_{c \in \mathbb{C}} \ \widehat{P}(c) \prod_{1 \leq k \leq n_d} \widehat{P}(t_k|c)$$

Taking the log

- Multiplying lots of small probabilities can result in floating point underflow.
- Since $\log(xy) = \log(x) + \log(y)$, we can sum log probabilities instead of multiplying probabilities.
- Since log is a monotonic function, the class with the highest score does not change.
- So what we usually compute in practice is:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} [\log \hat{P}(c) + \sum_{1 \leq k \leq n_d} \log \hat{P}(t_k|c)]$$

Naive Bayes classifier

- Classification rule:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} \, [\log \hat{P}(c) + \sum_{1 \leq k \leq n_d} \log \hat{P}(t_k|c)]$$

- Simple interpretation:
  - Each conditional parameter $\log \hat{P}(t_k|c)$ is a weight that indicates how good an indicator t_k is for c.
  - The prior $\log \hat{P}(c)$ is a weight that indicates the relative frequency of c.
  - The sum of log prior and term weights is then a measure of how much evidence there is for the document being in the class.
  - We select the class with the most evidence.

Parameter estimation take 1: Maximum likelihood

- Estimate parameters $\hat{P}(c)$ and $\hat{P}(t_k|c)$ from train data
- Prior:

$$\hat{P}(c) = \frac{N_c}{N}$$

  - N_c: number of docs in class c;
  - N: total number of docs
- Conditional probabilities:

$$\hat{P}(t|c) = \frac{T_{ct}}{\sum_{t' \in V} T_{ct'}}$$

  - T_{ct}: number of tokens of t in training documents from class c (includes multiple occurrences)

The problem with maximum likelihood estimates: Zeros

Document to classify:
Beijing and Taipei join the WTO

$$
\begin{aligned}
P(China|d) \quad \propto \quad & P(China) \cdot P(\textsc{Beijing}|China) \cdot P(\textsc{and}|China) \\
& \cdot P(\textsc{Taipei}|China) \cdot P(\textsc{join}|China) \cdot P(\mathrm{WTO}|China)
\end{aligned}
$$

- If WTO never occurs in class China in the train set:

$$\hat{P}(\mathrm{WTO}|China) = \frac{T_{China,\mathrm{WTO}}}{\sum_{t' \in V} T_{China,t'}} = \frac{0}{\sum_{t' \in V} T_{China,t'}} = 0$$

The problem with maximum likelihood estimates: Zeros (cont)

- If there are no occurrences of WTO in documents in class China, we get a zero estimate:

$$\hat{P}(\mathrm{WTO}|China) = \frac{T_{China,\mathrm{WTO}}}{\sum_{t' \in V} T_{China,t'}} = 0$$

- $\rightarrow$ We will get $P(China|d) = 0$ for any document that contains WTO!

To avoid zeros: Add-one smoothing

- Before:

$$\hat{P}(t|c) = \frac{T_{ct}}{\sum_{t' \in V} T_{ct'}}$$

- Now: Add one to each count to avoid zeros:

$$\hat{P}(t|c) = \frac{T_{ct} + 1}{\sum_{t' \in V}(T_{ct'} + 1)} = \frac{T_{ct} + 1}{(\sum_{t' \in V} T_{ct'}) + B}$$

- B is the number of bins – in this case the number of different words or the size of the vocabulary $|V| = M$

Naive Bayes: Summary

- Estimate parameters from the training corpus using add-one smoothing
- For a new document, for each class, compute sum of
  1. log of prior and
  2. logs of conditional probabilities of the terms
- Assign the document to the class with the largest score

Naive Bayes: Training

$\textsc{TrainMultinomialNB}(\mathbb{C}, \mathbb{D})$
$V \leftarrow \textsc{ExtractVocabulary}(\mathbb{D})$
$N \leftarrow \textsc{CountDocs}(\mathbb{D})$
for each $c \in \mathbb{C}$
do $N_c \leftarrow \textsc{CountDocsInClass}(\mathbb{D}, c)$
    $prior[c] \leftarrow N_c / N$
    $text_c \leftarrow \textsc{ConcatenateTextOfAllDocsInClass}(\mathbb{D}, c)$
    for each $t \in V$
    do $T_{ct} \leftarrow \textsc{CountTokensOfTerm}(text_c, t)$
    for each $t \in V$
    do $condprob[t][c] \leftarrow \frac{T_{ct}+1}{\sum_{t'}(T_{ct'}+1)}$
return $V, prior, condprob$

Naive Bayes: Testing

```
ApplyMultinomialNB(ℂ, V, prior, condprob, d)
W ← ExtractTokensFromDoc(V, d)
for each c ∈ ℂ
do score[c] ← log prior[c]
    for each t ∈ W
    do score[c] += log condprob[t][c]
return arg max_{c∈ℂ} score[c]
```

Exercise: Estimate parameters, classify test set

	docID	words in document	in $c = China$?
training set	1	Chinese Beijing Chinese	yes
	2	Chinese Chinese Shanghai	yes
	3	Chinese Macao	yes
	4	Tokyo Japan Chinese	no
test set	5	Chinese Chinese Chinese Tokyo Japan	?

$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(t|c) = \frac{T_{ct} + 1}{\sum_{t' \in V}(T_{ct'} + 1)} = \frac{T_{ct} + 1}{(\sum_{t' \in V} T_{ct'}) + B}$$

(B is the number of bins – in this case the number of different words or the size of the vocabulary $|V| = M$)

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} [\hat{P}(c) \cdot \prod_{1 \leq k \leq n_d} \hat{P}(t_k|c)]$$

Example: Parameter estimates

Priors: $\hat{P}(c) = 3/4$ and $\hat{P}(\overline{c}) = 1/4$
Conditional probabilities:

$$\begin{aligned}
\hat{P}(\textsc{Chinese}|c) &= (5+1)/(8+6) = 6/14 = 3/7 \\
\hat{P}(\textsc{Tokyo}|c) = \hat{P}(\textsc{Japan}|c) &= (0+1)/(8+6) = 1/14 \\
\hat{P}(\textsc{Chinese}|\overline{c}) &= (1+1)/(3+6) = 2/9 \\
\hat{P}(\textsc{Tokyo}|\overline{c}) = \hat{P}(\textsc{Japan}|\overline{c}) &= (1+1)/(3+6) = 2/9
\end{aligned}$$

The denominators are $(8+6)$ and $(3+6)$ because the lengths of $text_c$ and $text_{\overline{c}}$ are 8 and 3, respectively, and because the constant B is 6 as the vocabulary consists of six terms.

Example: Classification

$$\begin{aligned}
\hat{P}(c|d_5) &\propto 3/4 \cdot (3/7)^3 \cdot 1/14 \cdot 1/14 \approx 0.0003 \\
\hat{P}(\overline{c}|d_5) &\propto 1/4 \cdot (2/9)^3 \cdot 2/9 \cdot 2/9 \approx 0.0001
\end{aligned}$$

Thus, the classifier assigns the test document to $c = China$. The reason for this classification decision is that the three occurrences of the positive indicator Chinese in d_5 outweigh the occurrences of the two negative indicators Japan and Tokyo.

Time complexity of Naive Bayes

mode	time complexity
training	$\Theta(\lvert\mathbb{D}\rvert L_{\text{ave}} + \lvert\mathbb{C}\rvert\lvert V\rvert)$
testing	$\Theta(L_{\text{a}} + \lvert\mathbb{C}\rvert M_{\text{a}}) = \Theta(\lvert\mathbb{C}\rvert M_{\text{a}})$

- For training
  - L_{ave}: average length of a training doc,
  - $\mathbb{D}$: training set of documents,
  - $\Theta(|\mathbb{D}|L_{\text{ave}})$ is the time it takes to compute all counts.
  - V: vocabulary,
  - $\mathbb{C}$: set of classes
  - $\Theta(|\mathbb{C}||V|)$ is the time it takes to compute the parameters from the counts.
- For testing
  - L_{a}: length of the test doc,
  - M_{a}: number of distinct terms in the test doc,
- Generally: $|\mathbb{C}||V| < |\mathbb{D}|L_{\text{ave}}$
- Test time is also linear (in the length of the test document).
- Thus: Naive Bayes is linear in the size of the training set (training) and the test document (testing).

Outline

1. Text classification
2. Naive Bayes
3. NB theory
4. Evaluation of TC

Naive Bayes: Analysis

- Now we want to gain a better understanding of the properties of Naive Bayes.
- We will formally derive the classification rule ...
- ...and make our assumptions explicit.

Derivation of Naive Bayes rule

We want to find the class that is most likely given the document:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} P(c|d)$$

Apply Bayes rule $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} \frac{P(d|c)P(c)}{P(d)}$$

Drop denominator since $P(d)$ is the same for all classes:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} P(d|c)P(c)$$

Too many parameters / sparseness

$$\begin{aligned} c_{\text{map}} &= \underset{c \in \mathbb{C}}{\arg\max}\; P(d|c)P(c) \\ &= \underset{c \in \mathbb{C}}{\arg\max}\, P(\langle t_1, \ldots, t_k, \ldots, t_{n_d} \rangle | c) P(c) \end{aligned}$$

- There are too many parameters $P(\langle t_1, \ldots, t_k, \ldots, t_{n_d} \rangle | c)$, one for each unique combination of a class and a sequence of words.
- We would need a very, very large number of training examples to estimate that many parameters.
- This is the problem of data sparseness.

Naive Bayes conditional independence assumption

To reduce the number of parameters to a manageable size, we make the Naive Bayes conditional independence assumption:

$$P(d|c) = P(\langle t_1, \dots, t_{n_d} \rangle | c) = \prod_{1 \leq k \leq n_d} P(X_k = t_k | c)$$

We assume that the probability of observing the conjunction of attributes is equal to the product of the individual probabilities $P(X_k = t_k|c)$. Recall from earlier the estimates for these conditional probabilities: $\hat{P}(t|c) = \frac{T_{ct}+1}{(\sum_{t' \in V} T_{ct'})+B}$

Generative model

$P(c|d) \propto P(c) \prod_{1 \leq k \leq n_d} P(t_k|c)$

- Generate a class with probability $P(c)$
- Generate each of the words (in their respective positions), conditional on the class, but independent of each other, with probability $P(t_k|c)$
- To classify docs, we "reengineer" this process and find the class that is most likely to have generated the doc.

Second independence assumption

- $\hat{P}(X_{k_1} = t|c) = \hat{P}(X_{k_2} = t|c)$
- For example, for a document in the class *UK*, the probability of generating QUEEN in the first position of the document is the same as generating it in the last position.
- The two independence assumptions amount to the bag of words model.

A different Naive Bayes model: Bernoulli model

- term frequency is ignored
- good for short documents
- formula and algorithm are different
- ...

Violation of Naive Bayes independence assumptions

- Conditional independence:

$$P(\langle t_1, \ldots, t_{n_d} \rangle | c) = \prod_{1 \leq k \leq n_d} P(X_k = t_k | c)$$

- Positional independence:
- $\hat{P}(X_{k_1} = t | c) = \hat{P}(X_{k_2} = t | c)$
- The independence assumptions do not really hold of documents written in natural language.
- Exercise
  - Examples for why conditional independence assumption is not really true?
  - Examples for why positional independence assumption is not really true?
- How can Naive Bayes work if it makes such inappropriate assumptions?

Why does Naive Bayes work?

- Naive Bayes can work well even though conditional independence assumptions are badly violated.
- Example:

	c_1	c_2	class selected
true probability $P(c\|d)$	0.6	0.4	c_1
$\hat{P}(c) \prod_{1 \le k \le n_d} \hat{P}(t_k\|c)$	0.00099	0.00001	
NB estimate $\hat{P}(c\|d)$	0.99	0.01	c_1

- Double counting of evidence causes underestimation (0.01) and overestimation (0.99).
- Classification is about predicting the correct class and not about accurately estimating probabilities.
- Naive Bayes is terrible for correct estimation ...
- ...but if often performs well at accurate prediction (choosing the correct class).

Naive Bayes is not so naive

- Naive Bayes has won some bakeoffs (e.g., KDD-CUP 97)
- More robust to nonrelevant features than some more complex learning methods
- More robust to concept drift (changing of definition of class over time) than some more complex learning methods
- Better than methods like decision trees when we have many equally important features
- A good dependable baseline for text classification (but not the best)
- Optimal if independence assumptions hold (never true for text, but true for some domains)
- Very fast
- Low storage requirements

Outline

Example: The Reuters collection

symbol	statistic	value
N	documents	800,000
L	avg. # word tokens per document	200
M	word types	400,000

type of class	number	examples
region	366	UK, China
industry	870	poultry, coffee
subject area	126	elections, sports

A Reuters document

REUTERS

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Extreme conditions create rare Antarctic clouds

Tue Aug 1, 2006 3:20am ET

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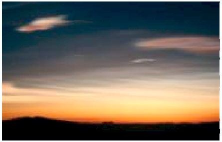

SYDNEY (Reuters) - Rare, mother-of-pearl colored clouds caused by extreme weather conditions above Antarctica are a possible indication of global warming, Australian scientists said on Tuesday.

Known as nacreous clouds, the spectacular formations showing delicate wisps of colors were photographed in the sky over an Australian

Evaluating classification

- Evaluation must be done on test data that are independent of the training data, i.e., training and test sets are disjoint.
- It's easy to get good performance on a test set that was available to the learner during training (e.g., just memorize the test set).
- Measures: Precision, recall, F_1, classification accuracy

Precision P and recall R

	in the class	not in the class
predicted to be in the class	true positives (TP)	false positives (FP)
predicted to not be in the class	false negatives (FN)	true negatives (TN)

TP, FP, FN, TN are counts of documents. The sum of these four

counts is the total number of documents.

$$\begin{aligned} \text{precision:}P &= TP/(TP + FP) \\ \text{recall:}R &= TP/(TP + FN) \end{aligned}$$

A combined measure: F

- F_1 allows us to trade off precision against recall.
- $$F_1 = \frac{1}{\frac{1}{2}\frac{1}{P} + \frac{1}{2}\frac{1}{R}} = \frac{2PR}{P+R}$$
- This is the harmonic mean of P and R: $\frac{1}{F} = \frac{1}{2}(\frac{1}{P} + \frac{1}{R})$

Averaging: Micro vs. Macro

- We now have an evaluation measure (F_1) for one class.
- But we also want a single number that measures the aggregate performance over all classes in the collection.
- Macroaveraging
  - Compute F_1 for each of the C classes
  - Average these C numbers
- Microaveraging
  - Compute TP, FP, FN for each of the C classes
  - Sum these C numbers (e.g., all TP to get aggregate TP)
  - Compute F_1 for aggregate TP, FP, FN

F_1 scores for Naive Bayes vs. other methods

(a)	NB	Rocchio	kNN		SVM
micro-avg-L (90 classes)	80	85	86		89
macro-avg (90 classes)	47	59	60		60

(b)	NB	Rocchio	kNN	trees	SVM
earn	96	93	97	98	98
acq	88	65	92	90	94
money-fx	57	47	78	66	75
grain	79	68	82	85	95
crude	80	70	86	85	89
trade	64	65	77	73	76
interest	65	63	74	67	78
ship	85	49	79	74	86
wheat	70	69	77	93	92
corn	65	48	78	92	90
micro-avg (top 10)	82	65	82	88	92
micro-avg-D (118 classes)	75	62	n/a	n/a	87

- Naive Bayes does pretty well, but some methods beat it consistently (e.g., SVM).

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